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Intransparent Markets and Intra-Industry Trade [*]

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Abstract:
Buyers are typically unaware of the full set of offers when making a purchase. This paper examines how international trade interacts with this problem of market intransparency. Sellers must communicate their offers through costly advertising, but cannot reach all buyers. Consequently, no market clearing price exists, and sellers randomize over an equilibrium price distribution. The model generates intra-industry trade that would not take place under complete information, bringing gains to buyers. Sellers in the model are identical, but appear heterogeneous due to their price randomization. If sellers differ slightly, these differences will be greatly magnified.

Keywords: advertising, intra-industry trade, firm heterogeneity, price dispersion

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1 Introduction

Information costs have long been thought to matter for international trade flows. Stylized facts of modern trade, such as the consistent findings of significantly higher trade flows between countries with a common language and the trade-promoting effect of ethnic networks, strongly support the hypothesis. To date, however, theoretical treatments have been sparse. Against this background, Anderson and Van Wincoop (2004, p720) conclude their review of the literature on information costs by stating that "More careful modeling of the underlying information costs in future work will probably be illuminating."

This paper takes up the task. The point of departure is the literature on how buyers and sellers match, a literature that accounts for the fact that many real world markets are "intransparent": The information necessary to carry out an exchange, such as who sells, what price these sellers charge and what the quality offered is, will not automatically reach buyers. Rather, sellers spend considerable resources on advertising their goods. Modeling intransparent markets and the associated advertising costs in an open economy setting uncovers intricate interactions between international trade and information costs. In particular, new leaves appear on two of the main current research branches in international trade: intra-industry trade and the role of firm heterogeneity in an open economy.

In the model, the positive mark-ups that encourage intra-industry trade are generated by advertising costs – if information could diffuse costlessly, entering a foreign market would not be profitable. Moreover, when markets are intransparent, different pricing strategies are equally profitable. Even though sellers are identical ex ante, both in their costs and in their ability to contact buyers, some will end up selling many units at low prices, others will charge high prices and sell fewer units. A hypothesis offered by this paper is therefore that some of the price-, sales- and export sales variations observed in firm-level data and typically attributed to differences in productivity may arise from similar firms following different strategies. Interestingly, introducing slight ex ante heterogeneity among sellers has dramatic consequences: If some sellers are slightly more productive than others, they will consistently undercut the less productive sellers, greatly exacerbating the ex ante heterogeneity.

I construct a model with two types of agents, buyers, who demand one unit of a good, and sellers, who produce it. Buyers are initially unaware of the characteristics of offers, and sellers must therefore advertise their offers. For expositional clarity, the focus is on homogeneous goods, where advertisement reduces to price posting, and where buyer search is assumed away. In this interpretation, the model is intended to describe industries where goods are relatively cheap (compared to the time cost of buyers), and where most buyers are primarily sensitive to price.[3] Relatively cheap goods imply that any gains for buyers of actively searching for offers, rather than passively evaluating offers received, are likely to be outweighed by time costs. As outlined in section 3.3, the modeling approach generalizes to quality-adjusted prices of non-homogeneous goods, where advertisement must transmit more information than the price, information that buyers cannot obtain through search. With this reinterpretation, the model covers any industry where goods are not tailored to fit individual buyers' particular tastes or needs.

The setup for the model builds on the wage posting model of Mortensen (1990, 2003). Sellers' advertising technology is similar to the seminal advertisement model of Butters (1977). Advertisement is non-rival in its form, one can think of sellers posting offers in mass media or in the public space.

[3] A few examples could be kitchen utensils, detergents, basic office equipment, and also copyrighted goods with multiple sellers, such as a particular book or recorded piece of music.

Even if sellers to some degree can segment buyers, this form of advertising inherently has some randomness to it. As a consequence, if there are many buyers, it becomes too expensive for the individual seller to reach them all. The randomness in advertising also leads to an ex post heterogeneity among buyers: Some buyers receive offers from multiple sellers and can select the best one, others receive no offer at all.

In this setting, there is no equilibrium price on the market: sellers will either want to price lower than other sellers, or to price higher, hoping that the buyer gets no better offer. The equilibrium outcome is a price distribution with no mass points, over which sellers randomize their price. Each seller thus charges a different price, although the good is homogeneous and sellers are identical. The price dispersion is sustained by unfortunate buyers, who, upon receiving only one expensive offer, have no better option for purchase. When realized, sellers' price randomization will result in different sales, mimicking price-quantity distributions from heterogeneous firms-models such as Melitz (2003), although sellers ex ante are entirely identical.

When sellers are able to contact buyers abroad, there will be two-way international trade in the model. The export market presents an entirely new set of buyers to sellers, and initially there is no risk of reaching the same buyer twice with the advertising campaign. The net implication of international trade is an increase in the average number of offers that a buyer learns about and a downward shift in the price distribution. International trade pushes the model towards the Bertrand equilibrium, to the benefit of buyers. Were it not for the information frictions, there would be no reason for international trade to occur, as both countries would be in Bertrand equilibrium already.

Associated with the information costs is therefore a new gain from international trade, a transparency gain: Buyers gain from receiving more information and from the subsequent intensified price competition. The closest parallel in the trade literature is the gain from trade put forward by the Cournot models of Brander (1981) and Brander and Krugman (1983), where welfare gains arise from the strategic responses of oligopolistic firms when the economy is opened. The gain from trade in the present model, however, remains with many sellers (a continuum).

International advertisement is likely to be easier between countries that share languages. Lower costs of export advertising will enable sellers to export more, the model thus presents an explicit channel for the well-established result that countries with shared languages trade more, see Melitz (2008) for a detailed empirical treatment.

Price dispersion, even for homogeneous goods or within specific brands, is a longstanding and consistent finding in economics. It has been documented empirically by, among others, Stigler (1961) and Pratt, Wise and Zeckhauser (1979); Clay et al. (2001) and Feenstra and Shapiro (2003) document that the phenomenon has not disappeared in the internet age. A rich theoretical literature has put forward different explanations for how price dispersion may occur, Butters (1977) and Burdett and Judd (1983) are seminal papers, see Baye, Morgan and Scholten (2006) for a recent review. To my knowledge, this paper is the first to make the obvious connection between price dispersion and intrinsic firm heterogeneity, the idea has been explored in labor economics, e.g. Mortensen (2003).

The two main results of this paper, that information costs may encourage international trade and create or magnify heterogeneous pricing strategies, differ markedly from previous theoretical works on the subject, many of which are very recent. Rauch (1999, 2001) and Rauch and Trindade (2002, 2003) sparked research into how business-, migrant - or social networks might facilitate international trade by spreading non-observable information on quality. The recent works of Chaney (2011a, 2011b) are very promising in showing how these (scale-free) network structures may be behind the specific functional form of the gravity equation, noticeably how distance enters.

Other key comparisons are Arkolakis (2010) and Eaton et al. (2011), which to my knowledge are the only other papers modeling advertisement (or "seller search") in international trade explicitly. Arkolakis (2010) shares a building block with this paper, advertisement costs in line with Butters (1977). The crucial difference is specification of demand; both Arkolakis (2010) and Chaney (2011a, 2011b) build on a love-of-variety model, Eaton et al. (2011) construct what we may call a "love-of-some-varieties" model. To draw lines sharply, the differences are as follows: Suppose a buyer receives advertisement about five different tumble-driers. In Arkolakis (2010) and Chaney (2011a, 2011b), the buyer would purchase all five, in Eaton et al. (2011), the buyer accepts all tumble driers that are exogenously put in his choice set. In the present paper the buyer selects the best of the five offers.[4] As we shall see, modifying demand will deeply enrich the set of possible predictions.

The next section sets up the model, thereafter section 2.1 shows how price dispersion arises. Section 2.2 analyzes why intra-industry trade occurs and what consequences it has, and how it compares to existing theories of intra-industry trade. Further implications of the model are analyzed in section 3: a comparison to the predictions of models with heterogeneous firms, a discussion of trade costs, and of quality differences. Concluding remarks follow in the final section 4.

2 Buyers, Sellers and Advertising Technology

Consider two countries, Home (H) and Foreign (F). In each country there is a fixed continuum of buyers, n^H and n^F respectively. Each buyer demands one unit of a homogenous good, and they all have a common reservation price of $\bar{p}$. This demand arises from a utility function of the form

$$U(p)=\begin{cases}\bar{p}-p & if \quad p<\bar{p}\\ 0 & otherwise\end{cases} \tag{1}$$

There is a continuum of sellers in each country, m^H and m^F. For the moment, let the number of sellers be fixed; in the next section I will analyze a free entry equilibrium where m^H and m^F are determined endogenously. Sellers produce the good at marginal cost c, with $c < \bar{p}$. Initially, buyers neither know individual sellers nor the prices they are charging for the good, and they are therefore unable to make a purchase. Sellers must inform buyers of their offers through advertising. The good is homogenous, and so the only information buyers need is the price; advertising reduces to price posting.

The costs of advertising consist of a fixed cost f_v of employing the relevant people and having them design the advertising campaign, and of variable costs of reaching domestic or foreign buyers. For a seller in Home, the cost of reaching k^H distinct domestic buyers is described by the function $v(k^H/n^H)$, satisfying the following assumption:

Assumption 1

(i) v is twice continuously differentiable, strictly increasing and strictly convex in k^H/n^H, $v'(k^H/n^H) > 0$ and $v''(k^H/n^H) > 0$.

(ii) $\lim_{k^H \to n^H} \dfrac{v'\left(k^H/n^H\right)}{n^H} > \left(\bar{p}-c\right)$

[4] Eaton et al. (2011) allow for the idea that the buyer's set of desired varieties may change. Anecdotal evidence provided by the authors indicates that some export buyer-seller matches indeed terminate because the buyer finds a better offer. The present paper directly models price competition between sellers. See also discussion in footnote 11.

The reasoning behind assumption 1 is that price posting is random, sellers cannot control exactly which buyers they reach. The campaign may hit the same buyer multiple times, and the larger the fraction of the population reached by the campaign, the higher the probability that resources will be wasted on reaching the same buyer twice. In the end, the cost of reaching the last buyer is higher than what the seller could potentially earn.[5]

If the seller in Home wishes to advertise her offers abroad, she will incur costs $v_x\left(k_x^H / n^F\right)$ of reaching k_x^H distinct buyers in Foreign. In addition to satisfying the convexity and limit conditions of Assumption 1, v_x also satisfies

Assumption 2

(i) $v_x(k/n) > v(k/n)$ for any k/n

(ii) $v_x'(k/n) > v'(k/n)$ for any k/n

(iii) $\lim_{k^H \to 0} v'_x\left(k_x^H / n^F\right) < f_v$

Cultural and language barriers, along with geographic distance make price posting abroad relatively more expensive, 2(i). Because a given campaign scope costs more on the export market, but faces a similar risk of reaching the same foreign buyer several times, export price posting costs also rise faster than their domestic counterpart, 2(ii). However, a seller can use some common resources for the foreign and domestic price posting campaigns. There may be a fixed cost of translating, modifying and launching the price posting campaign abroad, but it is lower than f_v, 2(iii).[6] Advertising costs for a seller in Foreign are completely analogous, just interchange superscripts H and F.

The structure of advertisement costs is similar to those in Butters (1977) and Arkolakis (2010). Because advertisements crowd each other out, there is decreasing returns to advertising. Stegeman (1991) and Arkolakis (2010) provide empirical evidence of decreasing returns to advertising, they may occur for other reasons than the increased risk of advertising to the same buyers twice, the exact mechanism is not important for the present paper.

The timing of the game is as follows: In the first stage, each seller chooses the scope of her domestic and foreign price posting campaigns, (k^H and k_x^H for a seller in Home) and her domestic and export prices p and p_x. In the second stage, each buyer picks the best among the offers he learns about. If a buyer only receives one offer, he buys the good if its price is not larger than the maximum willingness to pay; if there is more than one offer, the buyer will accept the cheapest offer. In case there are several offers with the lowest price, the buyer selects randomly among these. Buyers can only buy offers they learn about, and actively searching for goods is assumed to be too costly.[7]

To characterize the equilibrium, we need (i) a description of how many offers buyers receive and what prices these offers have, which is derived in the next section 2.1, (ii) to derive how sellers optimize their prices in both domestic and export markets such that no seller wishes to deviate, also in

[5] The price posting technology does not have to be completely random. The modeling is consistent with sellers splitting buyers into segments and advertising to some segments only. Buyers are homogenous, however: Sellers' choices of which segments to target are uncorrelated.

[6] Introducing additional per-unit trade costs into the model is possible, but cumbersome. For clarity, they are left out. A discussion is provided in section 3.2.

[7] A sufficient condition to rule out buyer search is that the expected cost of finding an offer through search is higher than $\bar{p} - c$. This condition is more likely to hold for relatively cheap goods. See also the discussion in footnote 14. Buyer search will return implicitly in section 3.3.

section 2.1, and (iii) sellers' optimal price posting scopes on domestic and export markets, derived in section 2.2. With these equilibrium behaviors established, section 2.2 proceeds with an analysis of how and why intra-industry trade may arise and of potential gains from trade. For brevity, I derive the model for buyers and sellers in Home only; equivalent results for Foreign can be obtained by exchanging superscripts *H* and *F*.

2.1 Price Randomization

Before solving the sellers' optimization problem, it is helpful to establish what the optimal pricing strategies must look like. Given the assumptions on advertisement technology, there is no equilibrium where some sellers set the same price. If a buyer receives several offers with the same price, a seller will always want to reduce her price slightly and be sure that the buyer accepts her offer rather than selects an offer at random. This undercutting does not continue, though: If all sellers were to price at c, a seller can earn positive profits by setting $p = \overline{p}$: the probability that the buyer gets no other offer is positive, since no seller contacts all buyers.

Sellers will therefore choose to randomize prices, both on the domestic and export markets. Prices in Home and Foreign will follow some cumulative distributions, call them $F^H(p)$ and $F^F(p)$. In Appendix A.1, I show that in equilibrium, these price distributions will be continuous and have connected support, with upper support $\overline{p}$ and lower support no less than c.

To determine the scope of the sellers' domestic and export price posting campaigns and the distributions over which they randomize prices, consider the expected profits that a seller in Home earns:

$$\Pi\left(p, k^H, p_x, k_x^H\right) = \pi\left(p, k^H\right) + \pi_x\left(p_x, k_x^H\right), \qquad (2)$$

where $\pi\left(p, k^H\right) = Q^H(p)(p-c)k^H - v\left(k^H / n^H\right) - f_v$,

and $\pi_x\left(p_x, k_x^H\right) = Q^F(p_x)(p_x - c)k_x^H - v\left(k_x^H / n^F\right)$

$\pi\left(p, k^H\right)$ is expected profits earned on the domestic market and $\pi_x\left(p_x, k_x^H\right)$ is profits earned abroad. $Q^H(p)$ is the probability that a buyer in Home accepts the offer at price p, and $Q^F(p_x)$ is the probability that a buyer in Foreign accepts the offer at export price p_x.

Sellers can treat the two markets as separate, the optimal choice of (p, k^H) does not affect the optimal choice of $\left(p_x, k_x^H\right)$. Moreover, I establish in Appendix A.2 that the optimal scopes of the price posting campaigns do not depend on what prices the seller charges, i.e. $k^H(p) = k^H$ and $k_x^H(p_x) = k_x^H$. This symmetry in price posting simplifies the derivation of the optimal mixed strategy in prices. Prices must be randomized in such a manner that all prices offered on a market must give the same expected profits. In particular, $\pi\left(p, k^H\right) = \pi\left(\overline{p}, k^H\right)$ and $\pi_x\left(p_x, k_x^H\right) = \pi_x\left(\overline{p}, k_x^H\right)$. These conditions reduce to

$$Q^H(p)(p-c) = Q^H(\overline{p})(\overline{p}-c) \qquad (3)$$

for the domestic market and

$$Q^F(p_x)(p_x - c) = Q^F(\overline{p})(\overline{p}-c) \qquad (4)$$

for the export market.

To derive $Q^H(p)$, the probability that a buyer in Home accepts an offer with price p, consider the number of offers a buyer in Home receives: The total number of offers a buyer receives, X, is the sum of offers from domestic and foreign sellers, $X = X^H + X^F$. Both X^H and X^F must be binomially distributed: Buyers are targeted at random, and the probabilities of being hit by a given domestic or foreign seller are k^H/n^H and k_x^F / n^F, respectively. There are m^H and m^F sellers making contacts, and so we have the two parameters for each of the two binomial distributions. As the sum of two binomial distributions with different base probabilities, the distribution of X is Poisson binomial.

Throughout this paper, I will assume that the crowding out of advertisement technology is sufficiently strong that sellers' optimal choices of k^H/n^H and k_x^F / n^F are small numbers, the distribution of X can be well approximated by a Poisson distribution[8]:

$$\Pr(X = x) = \frac{\exp(-\lambda^H)(\lambda^H)^x}{x!}, \text{ where } \lambda^H = \frac{m^H k^H + m^F k_x^F}{n^H}. \tag{5}$$

The Poisson parameter λ^H will be central to the model's results. It is equal to the expected number of offers a buyer receives, and as will be shown shortly, changes in λ^H will shift the distribution of prices. I will refer to λ^H as the contact frequency in Home.

I can now derive the distributions of prices, $F^H(p)$ and $F^F(p)$. There are two forces governing sellers' choice of price: The incentive to raise the price and earn a high mark-up"), hoping that the buyer does not get a better offer ("monopoly incentive), and the incentive to lower the price, increasing the probability of undercutting the other offers that a buyer receives ("Bertrand incentive"). The equilibrium distribution of prices offered is the distribution where these two incentives balance each other out. For a given price offer distribution, the probability that price p is the lowest among x other offers is $[1 - F^H(p)]^x$. Using this, the purchase probability $Q^H(p)$ can be computed as

$$\begin{aligned} Q^H(p) &= \sum_{x=0}^{\infty} \left[1 - F^H(p)\right]^x \frac{\exp(\lambda^H)(\lambda^H)^x}{x!} \\ &= \exp\left(-\lambda^H\left[1 - F^H(p)\right]\right) \sum_{x=0}^{\infty} \frac{\exp\left(-\lambda^H\left[1 - F^H(p)\right]\right)\left(\lambda^H\left[1 - F^H(p)\right]\right)^x}{x!} \\ &= \exp\left(-\lambda^H F^H(p)\right). \end{aligned} \tag{6}$$

Inserting (6) in (3) and using that $F(\overline{p}) = 1$ gives the distribution of prices that is consistent with sellers earning the same profits from all the prices they charge:

[8] The exact distribution for X, the Poisson binomial, is:

$$\Pr(X = x) = \sum_{i=0}^{x} \left(\frac{k^H}{n^H}\right)^i \left(\frac{k^H}{n^H}\right)^{m^H - i} \left(\frac{k_x^F}{n^H}\right)^{x-i} \left(1 - \frac{k_x^F}{n^H}\right)^{m^F - (x-i)} \frac{m^H!}{i!(m^H - i)!} \frac{m^F!}{(x-i)!(m^F - (x-i))!}$$

If k^H and k_x^F are not small relative to the number of buyers, as assumed throughout this paper, prices will be very close to the Bertrand equilibrium, as buyers then typically will have a large set of offers to select among. From equation (9) below, if all sellers increase k^H, they will lose profits because buyers then will have many offers to choose from, forcing them to cut back on advertisement. Since k^H does not vary across sellers, the assumption of k^H/n^H small is therefore quite natural.

$$\exp\left(-\lambda^H F^H(p)\right)(p-c) = \exp\left(-\lambda^H\right)(\bar{p}-c) \qquad (7)$$

$$\Leftrightarrow F^H(p) = 1 - \frac{1}{\lambda^H}\ln\left(\frac{\bar{p}-c}{p-c}\right) \qquad (8)$$

$F^H(p)$ has lower support $\exp\left(-\lambda^H\right)\bar{p} + \left(1-\exp\left(-\lambda^H\right)\right)$ and upper support $\bar{p}$. In equilibrium, sellers in Home randomize their domestic prices over $\left[\exp\left(-\lambda^H\right)\bar{p} + \left(1-\exp\left(-\lambda^H\right)\right), \bar{p}\right]$ in such a manner that prices offered will follow the distribution $F^H(p)$. If the contact frequency λ^H becomes very large, such that each buyer observes all prices offered, prices will approach the Bertrand equilibrium: The lower support tends to c, and $F(p) = 1$ for all $p > c$, all sellers would price at marginal cost.[9]

Export prices set by sellers in Foreign will also follow $F^H(p)$. The condition that Foreign's sellers' export prices must give the same profits, $\pi\left(p_x, k_x^F\right) = \pi\left(\bar{p}, k_x^F\right)$, also leads to (8).

The more offers buyers learn about on average (higher λ^H), the more the Bertrand incentive for sellers to undercut other offers will dominate, and prices will be lower stochastically. As long as there is a positive probability that some buyers only know one offer ex post, price dispersion can exist in equilibrium, even though the good is homogeneous. Buyers accepting unfavorable offers do not irrationally perceive these as superior, they simply do not know of any better offers. Sellers charging lower prices are not more efficient, they simply opt for the strategy of undercutting others, making up for lower margins through higher sales, but without earning more profits than high-price sellers.[10,11]

2.2 Intra-Industry Trade, Causes and Consequences

With the pricing behavior determined, we are now ready to examine whether intra-industry trade occurs and what consequences it has. If sellers in both countries set positive scopes for their export price posting campaigns (if $k_x^H > 0$ and $k_x^F > 0$) they will also occasionally make export sales, and there will be intra-industry trade. The optimal price posting scopes follow from the first order conditions for maximizing profits (2) with respect to k^H and k_x^H:

$$\frac{v'\left(k^H / n^H\right)}{n^H} = Q^H(p)(p-c)$$

$$\frac{v_x'\left(k_x^H / n^F\right)}{n^F} = Q^F(p_x)(p_x-c)$$

Inserting the expected mark-ups gives optimal price posting scopes as implicit functions of λ^H and λ^F:

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[9] The corresponding distributions for quantity and sales are presented in appendix A.4.

[10] Even though there is price dispersion in the economy, there is no room for arbitrage: A third party, buying the good at a price $p' > c$ with the purpose of resale would face the same information problem as the sellers and would have to perform price posting on his own. This third party would effectively correspond to a seller producing at a higher marginal cost, which is unprofitable relative to entering as a seller. This is also the reason why there can be no re-exporting from F to H of goods that are already exported from H to F.

[11] Butters (1977) provides a discussion of allowing for buyer search in a related framework. If search is not too costly, buyers will search if the offers they receive are all priced above a certain threshold p'. This will lead sellers never to price above p', the equilibrium now holding with p' replacing the reservation price.

$$\frac{v'\left(k^{H}/n^{H}\right)}{n^{H}}=\exp\left(-\lambda^{H}\right)\left(\bar{p}-c\right) \tag{9}$$

$$\frac{v_x'\left(k_x^{H}/n^{F}\right)}{n^{F}}=\exp\left(-\lambda^{F}\right)\left(\bar{p}-c\right) \tag{10}$$

The right-hand side of (10) is positive, so k_x^H must be positive for the equality to hold. As stated in Proposition 1, the model may therefore generate intra-industry trade.

> **Proposition 1:** If the costs of advertising abroad are not too large (in the sense defined below), there will be two-way trade in the homogenous good.

Sellers in Home will set $k_x^H > 0$ according to (10) if the costs of advertising abroad are lower than expected export revenue:

$$\frac{v_x\left(k_x^{H}/n^{F}\right)}{k_x^{H}}\leq\exp\left(-\lambda^{F}\right)\left(\bar{p}-c\right). \tag{11}$$

Or, inserting the first order condition (10),

$$\frac{v_x\left(k_x^{H}/n^{F}\right)}{k_x^{H}/n^{F}}\leq v_x'\left(k_x^{H}/n^{F}\right). \tag{12}$$

These two ways of expressing the conditions for positive export profits point to two reasons why advertising abroad may not be worthwhile. Either advertising abroad may simply be too inefficient: at the optimal scope, the average cost is higher than the marginal cost, if (12) does not hold. Or, if (11) fails, the contact frequency in Foreign may already be so high that there is not enough export revenue to earn.

The relationship between information costs and international trade is intricate: Mark-ups will only be high enough to encourage intra-industry trade if it is difficult for domestic sellers to widely diffuse information about their offers. If it were costless to communicate offers, there would be no reason for the two countries to trade. On the other hand, information costs may of course discourage international trade if it is very costly to communicate offers abroad. Finally, the micro-structure of information costs matter: If advertising costs increase very steeply when sellers try to reach more buyers (if v is "very convex"), international trade becomes more likely because it then is cheaper to contact a few buyers at home and a few buyers abroad, rather than concentrating advertisement in one market.

The motivation for intra-industry trade is most similar to the one in Cournot models of trade, introduced by Brander (1981) and Brander and Krugman (1983). In both types of models, competition may be low enough in autarky to encourage intra-industry trade, even for homogenous goods. In the present model, price competition is low if information costs are high; in the Cournot models, if only a few domestic firms are present in a market, quantity competition is low, encouraging foreign firms to enter the market.

Moreover, a condition for intra-industry trade in both models is that markets are segmented. In this paper, market segmentation means that the export market presents sellers with a whole new set of buyers. Initially, sellers face no risk of wasting resources by hitting the same foreign buyers twice, and they can advertise to these foreign buyers without crowding out their domestic advertisement. On the margin, beginning to advertise abroad is therefore more efficient than advertising at home. In Brander (1981) and Brander and Krugman (1983), market segmentation means that firms can set their export quantities without crowding out domestic demand. The present paper extends the Cournot models in one important direction: Intra-industry trade may exist for homogeneous goods even if there are many sellers (the Brander and Krugman (1983) story is usually framed as "there are too few firms in autarky to drive mark-ups down", even though formally the model contains *n* firms).

Proposition 2 outlines the consequences of intra-industry trade:

> **Proposition 2:** Two-way trade will lower prices stochastically and increase the buyer surplus relative to autarky. If there is free entry to become a seller, two-way trade will improve total welfare.
> Proof: See Appendix A.3.

International trade drives up the contact frequencies, λ^H and λ^F. Buyers gain both directly, because they on average get more offers to choose from, and indirectly, because the higher contact frequencies increase the incentive for sellers to undercut each other, leading to lower prices. That buyers gain from lower prices is a quite typical finding in models of international trade, but the additional gain from having a larger set of offers to choose from is novel. International trade improves the information set available to buyers when they make a purchase, we could call this gain from trade a "transparency gain."[12]

It follows quite naturally that international trade improves total welfare when there is free entry: Free entry will drive sellers' profits to zero both in autarky and in the open economy (all revenue will be spent on advertisement). Welfare is then simply equal to the buyer surplus, which increases.

3 Discussions and Perspectives

After a discussion just below of the welfare gains in the model, section 3.1 relates the model to firm heterogeneity, section 3.2 discusses per-unit trade costs and in section 3.3 I introduce vertical quality differences.

3.1 The Transparency Gain

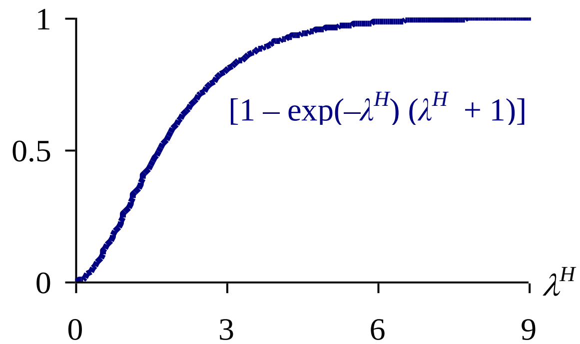
As shown in Appendix A.3, buyer surplus, accruing to buyers that pay less than their reservation price $\bar{p}$ can be expressed as

$$BS^H = n^H(\bar{p}-c)\left[1-\exp\left(-\lambda^H\right)\left(\lambda^H+1\right)\right]$$

[12] There is another model of intra-industry trade in a homogeneous good Bertrand setting, due to Cukrowski and Aksen (2003). Trade is here driven by uncertain demands in both markets and brings with it a "diversification gain", as risk-averse firms can reduce their risk exposure by serving both the domestic and the export markets. As sellers' realized sales in my model are stochastic, serving both markets also brings reduced revenue variance. Being risk-neutral, however, sellers do not value this. Stretching the interpretation of their model a bit, Cukrowski and Aksen (2003) show a result regarding incomplete information similar to this model: The driver of intra-industry trade is incomplete information, and, as in my model, improved flows of domestic market information is a detriment to trade.

The term [1 – exp(–λ^H)(λ^H + 1)] $\in$ [0,1) measures how far the buyer surplus is from its Bertrand level, $n^H(\bar{p}-c)$. This "intransparency loss" from incomplete information is depicted in Figure 1:

Figure 1: The intransparency loss



Buyer surplus as function of the contact frequency. On the y-axis, 1 corresponds to buyer surplus under Bertrand competition.

When the closed economy value of λ^H is small, even if international trade only brings a modest increase in λ^H, the resultant transparency gain is high. On the other hand, the new motive for intra-industry trade outlined in this paper is not omni-present: When buyers already have good information on sellers' offers, there is little revenue for potential foreign sellers to reap, and export price posting may not take place at all.

Trade may be facilitated through lower cost of price posting abroad, represented by a downward shift in $v_x\left(k_x^H / n^F\right)$. For illustration, suppose export price posting costs fall proportionally, to $\alpha v_x\left(k_x^F / n^H\right)$, with $0 < \alpha < 1$. The first order condition for k_x^H is changed to

$$\alpha v_x'\left(k_x^H / n^F\right) = \exp\left(-\lambda^F\right)\left(\bar{p}-c\right)$$

which implies an increase in export price posting. The net implication will be an increase in λ^F, to the benefit of the buyers in Foreign.

This comparative statics exercise provides an additional insight: It is plausible that in countries sharing a language or having similar cultures, foreign price posting costs v_x will be closer to domestic costs v, and therefore trade and the gains thereof will be higher. The analysis of this paper thus presents an explicit channel for the well-known empirical result that countries with similar languages trade more, see for instance Melitz (2008).

The IT revolution of the last two decades has provided sellers with a cheap price posting device, which does not require any physical proximity to buyers. In terms of the model, the ascent of the internet represents a reduction in both v and v_x, with the reduction in v_x likely being more pronounced. It is not certain, however, that the internet will promote trade. Comparative statics allow for the possibility that the drop in v is sufficient to remove the motive for export price posting. However, if v_x drops from a prohibitive level and approaches v, intra-industry trade is likely to increase. This pattern seems to apply to the markets for books, music and movies, which prior to the internet were dominated by local retail sales, but where consumers now often import the goods directly themselves.

Some markets, on the other hand, have institutions that ensure full information to buyers, notably the futures exchanges where many commodities, such as unprocessed metals and the major crops, are sold. The model presented outlines one of the benefits of such institutions: they remove buyers' intransparency loss and the need to for sellers to spend resources on price posting.

3.2 Heterogeneous Firms or Heterogeneous Strategies?

One of the most prominent frameworks for modeling international trade flows is models with monopolistic competition and heterogeneous firms; Melitz (2003) is the seminal paper. In these models, firms differ in their marginal costs, and therefore also in their prices and sales. Only the most productive firms are able to cover the additional costs of exporting, and international trade will drive out the least productive firms.

An interesting feature of the model presented in this paper is that it replicates many of the predictions of models with heterogeneous firms, even though sellers are completely homogenous ex ante. Because sellers randomize their prices, there is a whole realized distribution of sellers with different price-quantity combinations. In one extreme, a seller sets a price of $\bar{p}$ and sells an expected quantity of $\exp(-\lambda^H)k^H$, the other extreme is a seller setting a price of $\exp(-\lambda^H)\bar{p}+(1-\exp(-\lambda^H))c$ selling expected quantity of k^H: Homogenous firms following heterogeneous strategies.

Sellers all expect the same profit on the export market, but sellers setting higher export prices export less in expected terms and are more likely not to carry out any export sales at all. The pattern is again similar to models of trade with heterogeneous firms, where less productive firms tend to export less or not at all. Moreover, the model rationalizes a feature of firms' export behavior that these models have a hard time catching: As laid out in Eaton, Eslava, Krugler and Tybout (2007), many firms export only small quantities and very infrequently. Such behavior is peculiar if these firms have invested large resources in exporting, but it comes out naturally in the present model: Once in a while, sellers with high export prices are lucky enough to reach a foreign buyer who has no better offer available.

I do not mean to suggest that the large differences across firms that we observe in firm-level data arise only because of price randomization. The evidence that firms actually differ in their underlying characteristics is overwhelming (despite the difficulties in precisely measuring firms' productivities). It is, however, an intriguing hypothesis that some of the heterogeneity and variations in international activity observed in the firm-level data may arise from similar firms following different, but equally profitable strategies, especially in light of papers showing that productivity differences cannot explain all the variation in firms' export performance, see Munch and Nguyen (2010) and Armenter and Koren (2009).

To develop further on this point, consider how price dispersion interacts with true heterogeneity. Imagine we have two types of sellers, with marginal costs c_1 and c_2, with c_1 being just marginally larger than c_2. Type-1 sellers will have a larger incentive to undercut competing offers than type-2 sellers. Conversely, type-2 sellers have a larger incentive to charge a high price and hope that contacted buyers do not get a better offer.

The result will be a split in the price distribution, with type-2 sellers pricing in the upper part of the distribution and type-1 sellers in the lower part, even though the underlying difference in marginal costs is very small. Incomplete information can therefore both greatly exacerbate existing differences between firms and generate new differences by allowing identical firms to follow different strategies. The result generalizes to any countable number of seller types.

One of the main puzzles with models of heterogeneous firms is how these large differences in firms' productivities can persist over time. Why do the unproductive firms not just copy the technology of the productive firms? A hypothesis offered here is that intrinsic differences between firms might actually be quite small, but that these small differences radically change firms' optimal strategies.

3.3 Trade Costs

Per unit trade costs have been left out so far in order to simplify the exposition. Conceptually, they fit in nicely as supplementary costs of delivering the good to foreign buyers. Per-unit trade costs produce the same "knife-edge" prediction as small differences between sellers' marginal costs in the previous section. With their increased marginal costs, exporting sellers have weaker incentives to undercut other sellers than their domestic competitors. Just as for sellers with different costs, the equilibrium price distribution will split in two, with foreign sellers pricing in the upper part of the distribution and domestic sellers in the lower part. This prediction, although somewhat extreme, is actually more in line with firms' export pricing than the customary iceberg cost of exporting: Firms raise their f.o.b. price when exporting to distant markets.[13]

A basic property of models with heterogeneous firms is carried over if exporters incur higher marginal costs: Exporters still need to be special to perform as well as the domestic firms on their destination market, for example by having lower marginal costs.

With trade costs included, the modeling approach of this paper could be used to augment the realism at the micro-level of the canonical models of Eaton and Kortum (2002) and Eaton, Bernard, Jensen and Kortum (2003). In these models, all trade is one-way at the industry level; countries only source a good from the most efficient producer (adjusted for trade-costs). This prediction is at odds with the data. Adding a layer of intransparency at the goods-level would allow countries to have intra-industry trade, even though one country has superior technology.

3.4 Quality Differences

So far I've considered only homogenous goods. We have seen that intra-industry in homogenous goods may exist even if there are many sellers, and that these sellers can be heterogeneous in strategies. Generalizing the model to allow for vertical quality differences is quite straightforward.[14] The aim here is not to develop a full-fledged model, but to show how it may be constructed. I therefore do not dwell on why sellers produce at different quality levels; they simply do so, for some exogenous reason. I consider a fixed continuum of sellers rather than free entry.

The model with vertical quality differences fits a much larger set of industries, now including for example domestic appliances, household electronics, computers and other office machinery, as well as standardized production machinery and intermediates (common chemicals, for instance). The key

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[13] The iceberg cost assumption is that for one unit to arrive at the export destination, τ>1 units must be shipped, the remaining τ–1 units "melt away" during transit. This assumption leads to the erroneous prediction that firms' f.o.b. prices will either be lower on the export market or equal to the domestic price. See Martin (2010) for derivations and empirical documentation.

[14] Incorporating horizontal quality differences, where buyers have to learn where to find their ideal variety, is a more complicated endeavor. Since the equivalence between ideal variety and love-of-variety models break down in Arkolarkis (2010) and Eaton et al. (2011), it might well be valuable to learn exactly what varieties buyers are getting when sellers must search or advertise, but such explorations are beyond the scope of the present paper.

characteristics of such industries are: Goods are not tailored to fit a particular buyer's taste or needs; goods may represent a significant share of a buyer's income; and goods are to some degree experience goods, quality differences are typically hard to assess completely before purchase.

The broader interpretation of prices also calls for a reinterpretation of buyer behavior and advertising activities. Since these goods are relatively expensive for the buyers, it is worthwhile for the buyers to actively search for offers. The time costs of searching for offers on light bulbs or tumble dryers are presumably more or less equal, but the cost of buying the first, the best light bulb is much lower. A natural assumption is therefore that in a ‘quality-adjusted price’-interpretation of the model, all sellers' prices are known to buyers. What buyers cannot know is their reservation price of a particular offer, and this is why sellers must advertise.

Advertising in these industries often contains no information on price. The literature on experience goods and signaling, with Nelson (1970, 1974) and Milgrom and Roberts (1986) as key contributions, has interpreted advertising in this context as credible signals of quality characteristics that buyers cannot verify before purchase. Reinterpreting for this model, advertising would now enable buyers to determine their willingness to pay for a particular offer. If a buyer sees an advertisement from only one seller, he will purchase this seller's good if the sales price is lower than his newly computed willingness to pay. If he sees several advertisements, he will purchase the good from the seller that offers the highest difference between reservation price and sales price.

A necessary assumption to retain the predictions of the homogenous good case is that the cost of quality is proportional to its value: When a good has quality θ, it implies that the buyer's willingness to pay for the good is $\theta\bar{p}$ and that the marginal cost of producing the good is θc. A seller producing with quality θ has expected profits of

$$\begin{aligned}\pi\left(p,k^H,p_x,k_x^H,\theta\right) = {} & Q^H(p,\theta)(p-\theta c)k^H + Q^F(p_x,\theta)(p_x-\theta c)k_x^F \\ & - v\left(k^H/n^H,\theta\right) - v_x\left(k_x^F/n^F,\theta\right) - f_v(\theta)\end{aligned}$$

where the costs of advertising may vary with quality.

A buyer selects the offer observed that offers the lowest quality adjusted price, defined as $p/\theta\bar{p}$. The purchase probability can be derived as in (6), $Q^H(p,\theta) = \exp(-\lambda^H F(p/\theta))$, and sellers will still randomize their price, with $\theta\bar{p}$ as the highest price charged. The condition that any price, which a seller with quality θ charges, must make the same expected profits as charging $\theta\bar{p}$ is

$$\exp\left(-\lambda^H F(p/\theta)\right)(p-\theta c) = \exp\left(-\lambda^H\right)(\theta\bar{p}-\theta c).$$

The cumulative distribution of quality-adjusted prices follows:

$$F^H(p/\theta) = 1 - \frac{1}{\lambda^H}\ln\left(\frac{\bar{p}-c}{p/\theta - c}\right). \qquad (13)$$

The optimal pricing strategy is to raise the price proportional to the quality of the good and then randomize according to (13). International trade increases λ^H, pushing the price/quality trade-offs offered to buyers downwards, with prices approaching the marginal cost of quality.[15]

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[15] The optimal number of buyers to advertise to will vary with the quality offered, but does not vary with the quality-adjusted price charged.

It is interesting in its own right that quality must be "welfare insensitive" for the price randomization predictions to carry over. If the cost of producing quality were not proportional to the buyers' valuation of quality, high quality sellers would have an incentive to undercut other sellers (if $c'(\theta) < \bar{p}'(\theta)$) or to set higher quality-adjusted prices (if $c'(\theta) > \bar{p}'(\theta)$). Just as for differences in marginal costs or per-unit trade cost, the price distribution would split. Buyers could then infer quality directly from the price range, obliterating the need of advertising to signal quality! These fascinating links between the informative and the signaling motive for advertisement warrant future investigation.

If there were per-unit (not ad valorem) trade costs in the open economy, the model would generate Alchian-Allen effects, it might only be worthwhile to export high-quality goods. If quality is "welfare insensitive", the expected mark-up earned per unit exported would be $\exp\left(\lambda^F\right)\left(\bar{p} - c - t/\theta\right)$, which is clearly increasing in θ. High-quality exported goods have a higher probability of 'hiding' among other high-priced goods; the effect may persist even if producing quality is costlier than buyers' valuation of it.

Naturally, one can think of more sophisticated purchase processes, with buyers inferring or learning the good's quality through repeated exposure to advertisement, peer effects or observed sales. Advertisement may spill over across borders, perhaps enabling some sellers to create an international brand for their good. This paper hopefully provides a framework for further work on these issues.

4 Conclusion

This paper proposes an entirely new trade model, with the literature on price dispersion as building blocks. From this alternative model emerges a whole new appreciation of information frictions. In international trade, we typically think of information frictions as an impediment to international trade, something that we can roughly model with iceberg costs or control for in gravity regressions using various proxies. The analysis presented above clearly demonstrates that information friction have much richer implications.

By reducing competition, information frictions may motivate international trade rather than hinder it, generating intra-industry trade that persists even if there are many sellers of a homogeneous product. Moreover, understanding information frictions provides radically new perspectives on firm heterogeneity: Identical sellers may follow a whole range of different strategies if they face uninformed buyers; and if sellers differ ever so slightly, these differences will be greatly magnified.

With the increasing availability of scanner data (see Feenstra and Shapiro (2003)) and transaction-level trade data (see Eaton, Eslava, Krugler and Tybout (2007) and Eaton et al. (2011)), the prospects for refining the model in interaction with data are good. Another interesting estimation strategy would be to simply survey buyers directly, obtaining a direct estimate of the key parameter of interest by asking them how many other offers they were considering before making a purchase.

I hope that this paper offers a fruitful starting point for new theoretical and empirical analysis of why and how we observe firms as heterogeneous and of the role of price competition and advertisement in international trade, all while adding to the burgeoning literature on information barriers and exporter behavior.

A final point of the paper is to highlight the limitations of sticking with just market structure, that of monopolistic competition. Neary (2003) has forcefully argued for considering alternatives, in particular strategic behavior of firms, and this paper lends additional force to his arguments. It is an important insight that price competition is consistent with intra-industry trade: Bernard, Eaton, Jensen and Kortum (2003) could be extended with the methods of this paper, allowing for intra-industry trade, in better accordance with the data. As demonstrated in Blas and Russ (2011), Bertrand pricing can lead to greatly magnified welfare gains of trade.

Appendix A.1: *Proof that $F^H(p)$ is continuous, has connected support, with upper support of $\bar{p}$ and lower support no less than c.*

These properties of the distribution of prices are equivalent to proposition 1 in Mortensen (2003), and the proof is adapted from there.

Continuity of $F^H(p)$ implies that the distribution has no mass points. Therefore, there is no pure strategy equilibrium where all sellers offer the same price. To see this, first observe that if all sellers offer the same price, the probability that a buyer accepts the seller's offer among x other offers is $1/(1+x)$. The probability of making a sale is therefore

$$Q^H(p) = \sum_{x=0}^{\infty}\left(\frac{1}{1+x}\right)\frac{e^{-\lambda}\lambda^x}{x!} = \frac{1}{\lambda}\sum_{x=0}^{\infty}\frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!} = \frac{1}{\lambda}\sum_{x=1}^{\infty}\frac{e^{-\lambda}\lambda^x}{x!} = \frac{1-e^{-\lambda}}{\lambda} < 1$$

(The result, proven below that all sellers contact the same number of buyers also holds in this case).

Therefore, a seller can do strictly better by decreasing her price with ε and being certain that her offer is accepted, $k^H(p-\varepsilon-c) > k^H Q^H(p)(p-c)$ for ε sufficiently small. If all sellers were to offer $p = c$, one seller could instead offer $p = \bar{p}$ and earn positive expected profits, since the probability that this is the only offer a buyer receives is $\exp(-\lambda^H) > 0$.

A similar argument rules out any equilibrium where some strictly positive fraction of sellers set the same price, establishing continuity of $F^H(p)$. Connectedness follows from the fact that a gap, say between p' and p'', with $p'<p''$, would lead to the contradiction $\pi(p,F^H(p)) > \pi(p',F^H(p'))$ for all $p\in(p', p'']$, since $F^H(p') = F^H(p'')$.

The upper support must be equal to $\bar{p}$: if a seller is certain that no higher price will be posted, she can only sell the good if the buyer receives no other offer. If a buyer receives no other offer, the seller earns the most by offering $p = \bar{p}$: $\arg\max_{p\le\bar{p}} \pi(p,k) = \arg\max_{p\le\bar{p}} ke^{-\lambda}(p-c) = \bar{p}$.

It is never profitable to offer a price lower than c, as long as sellers make non-negative profits, the lower bound will be larger than c.

Appendix A.2: *Proof that $k^H(p) = k^H$, and all $k_x^H(p) = k_x^H$ sellers contact the same number of buyers*

The first order conditions for maximizing domestic profits $\pi(p,k^H)$ (in (2)) with respect to k^H is

$Q(p)(p-c) = v'(k^H/n^H)/n^H$

Any (p, k^H) pair offered must give the same profit. Specifically, it must hold that $\pi(p,k^H) = \pi(\bar{p},\bar{k})$, where $\bar{k}$ is the optimal number of contacts when offering $\bar{p}$. Inserting the first order condition into this equality gives

$$v'(k^H/n^H)(k^H/n^H) - v(k^H/n^H) = v'(\bar{k}/n^H)(\bar{k}/n^H) - v(\bar{k}/n^H).$$

Differentiating the left-hand side with respect to k^H/n^H gives $v''(k^H/n^H)(k^H/n^H)$, which is positive when $k^H/n^H > 0$. The function $f(k^H/n^H) = v'(k^H/n^H)(k^H/n^H) - v(k^H/n^H)$ is monotonously increasing for

k^H/n^H positive. It follows that $k^H(p) = k^H$: In equilibrium, all sellers contact the same number of buyers, and this optimal number of contacts does not depend on the prices they offer. The proof that $k_x^H(p) = k_x^H$ is completely analogous.

Appendix A.3: *Proof of Proposition 2*

Proving Proposition 2 requires showing: i) The contact frequencies increase when the economy is opened. ii) The buyer surplus increases with the contact frequency. iii) Result (i) also holds when the number of sellers adjusts because of free entry.

i) Proof that going from autarky to the open economy increases contact frequencies in both countries.

Consider first the case where the numbers of sellers, m^H and m^F, do not change when the economy is opened. Contact frequencies in Home and Foreign are given by

$$\lambda^H = \frac{m^H k^H + m^F k_x^F}{n^H} \text{ and } \lambda^F = \frac{m^F k^F + m^H k_x^H}{n^F}, \qquad (5)$$

In (5), λ^H increases with k^H.

Conversely, k^H is defined implicitly from the first order condition (9), as a decreasing function of λ^H:

$$\frac{v'\left(k^H / n^H\right)}{n^H} = \exp\left(-\lambda^H\right)\left(\bar{p} - c\right). \qquad (9)$$

Imagine k_x^F jumps from zero to a positive value. If λ^H does not increase because of this jump, it will lead to a contradiction:

From (5), λ^H will increase, unless k^H decreases sufficiently to offset the change in k_x^F. But from the first order condition, domestic sellers would only lower k^H if λ^H increases. Hence, λ^H must increase when trade is opened. Similarly, Foreign's contact frequency must also rise if k_x^H jumps to a positive value.

Because the price distribution $F^H(p)$ shifts downward with λ^H, prices will be stochastically lower in Home, when the economy is opened.

ii) Proof that the buyer surplus increases with the contact frequency.

"Buyer surplus" accrues to buyers that pay less than their reservation price $\bar{p}$, as described by their utility function (1). Buyers receiving no offers are equivalent to buyers paying $\bar{p}$. By the law of large numbers, aggregate buyer surplus, *BS*, is equal to the sum of each buyer's expected utility:

$$BS^H = n^H\left[\bar{p} - E_b{}^H(p)\right]$$

where $E_b{}^H(p)$ is the price each buyer can expect to pay ex ante, before any price posting takes place.

I will now show that $E_b{}^H(p) = c + \exp\left(-\lambda^H\right)\left(\bar{p} - c\right)\left(\lambda^H + 1\right)$. To avoid notational clutter, I will drop the Home superscripts on all variables; the derivations hold for both Home and Foreign.

The purchase probability, $Q(p)$, calculated in (7) denotes the probability that all offers that a buyer receives have prices equal to or greater than p. The complementary event that at least one price is lower than p has probability

Pr(at least one offer has price lower than p)

$$= 1 - Q(p) = 1 - e^{-\lambda}\,\frac{\bar{p}-c}{p-c}$$

The probability of receiving no offer is equal to exp($-\lambda^H$).

If the buyer has received an offer lower than p, it means that the price he actually paid for the good, call it p_{paid}, is no lower than p:

$$\Pr\left(p_{paid} \leq p\right) = 1 - e^{-\lambda}\,\frac{\bar{p}-c}{p-c}$$

This probability gives the cumulative distribution of the price buyers pay, call it $F_b(p)$:

$$F_b(p) = 1 - e^{-\lambda}\,\frac{\bar{p}-c}{p-c}.$$

As buyers getting no offers receive no buyer surplus and therefore in welfare terms are equivalent to buyers paying $\bar{p}$, the cumulative distribution has mass point $\Pr\left(P = \bar{p}\right) = e^{-\lambda}$. The corresponding density is given by

$$f_b(p) = 1 - e^{-\lambda}\,\bar{p} - c\,\frac{1}{(p-c)^2}\text{, and } f_b(\bar{p}) = e^{-\lambda}.$$

$E_b(p)$ can now be computed:

$$E_b(p) = \int_{e^{-\lambda}\bar{p}+\left(1+e^{-\lambda}\right)c}^{\bar{p}} p f_b(p)\,dp + e^{-\lambda}\,\bar{p} = e^{-\lambda}\left(\bar{p}-c\right)\int_{e^{-\lambda}\bar{p}+\left(1+e^{-\lambda}\right)c}^{\bar{p}} \frac{p}{(p-c)^2}\,dp + e^{-\lambda}\,\bar{p}.$$

Integrating by parts gives:

$$\begin{aligned}
E_b(p) &= e^{-\lambda}\left(\bar{p}-c\right)\left[\frac{-\bar{p}}{\bar{p}-c} - \left(e^{-\lambda}\,\bar{p} + \left(1+e^{-\lambda}\right)c\right)\frac{(-1)}{e^{-\lambda}\left(\bar{p}-c\right)}\right] \\
&\quad - \int_{e^{-\lambda}\bar{p}+\left(1+e^{-\lambda}\right)c}^{\bar{p}} \frac{p}{(p-c)^2}\,dp + e^{-\lambda}\,\bar{p}. \\
&= e^{-\lambda}\,\bar{p} + \left(1-e^{-\lambda}\right)c - e^{-\lambda}\,\bar{p} + e^{-\lambda}\left(\bar{p}-c\right)\left[\ln\left(\bar{p}-c\right) - \ln\left(e^{-\lambda}\left(p-c\right)\right)\right] + e^{-\lambda}\,\bar{p} \\
&= c + e^{-\lambda}\left(\bar{p}-c\right)\left(\lambda+1\right)
\end{aligned}$$

With this expression for $E_b^H(p)$, buyer surplus is:

$$BS^H = n^H\left(\bar{p}-c\right)\left(1 - \exp\left(-\lambda^H\right)\left(\lambda^H+1\right)\right)$$

Buyer surplus is an increasing function of λ^H for $\lambda^H \geq 0$. Opening the economy therefore increases buyer surplus.

iii) Proof that going from autarky to an open economy increases welfare when there is free entry for becoming a seller.

When there is free entry, new sellers will enter until each seller's profit is driven to zero. In autarky, when $\pi_x = 0$, sellers earn zero profits when

$$e^{-\lambda}(\bar{p} - c)k = v(k/n) + f_v; \qquad \text{(A1)}$$

the condition holds in both countries, autarky variables are without country superscript.

Combining this zero profit condition with the optimality condition for *k*, (9), one gets

$$v'(k/n) = \frac{v(k/n) + f_v}{k/n} \qquad \text{(A2)}$$

Under free entry, the autarky price posting scope *k/n* must be where the average cost of price posting is minimized, this happens where the marginal price posting cost equals the average price posting cost. The fraction of buyers reached by the individual seller is therefore determined uniquely by price posting technology.

Expression (A2) is the motivation for including a fixed cost of advertising: If it weren't for f_v, free entry would make *k/n* arbitrarily small. The free entry level of *k/n* is therefore the lowest price posting scope that the economy can sustain, given that sellers must earn non-negative profits.

With the price posting scope determined by (A2), the autarky contact frequency λ that prevails under free entry may be found from (9) as

$$\lambda = \ln\left(\frac{n(\bar{p} - c)}{v'(k/n)}\right), \qquad \text{(A3)}$$

the contact frequency decreases with the price posting scope of each individual seller. Since $\lambda = mk/n$, the number of sellers under free entry is

$$m = \frac{n}{k}\ln\left(\frac{n(\bar{p} - c)}{v'(k/n)}\right), \qquad \text{(A4)}$$

which also decreases with *k/n*. Because *k/n* under free entry is at the lowest level the closed economy can sustain, the contact frequency, the number of sellers and buyer surplus cannot be higher in autarky than when there is free entry.

In the open economy, setting profits equal to zero for a seller from Home and inserting the first order conditions (9) and (10) gives:

$$k^H/n^H = \frac{v(k^H/n^H) + f_v + \left(v_x(k_x^H/n^F) - v_x'(k_x^H/n^F)(k_x^H/n^F)\right)}{v'(k^H/n^H)}. \qquad \text{(A5)}$$

By the convexity of $v_x(k_x^H/n^F)$, the term $v_x(k_x^H/n^F) - v_x'(k_x^H/n^F)(k_x^H/n^F)$ is negative, so, comparing to (11), k^H/n^H decreases when the economy is opened. Compared to autarky, sellers reallocate resources from domestic to export price posting and reach fewer buyers on the domestic market.
The contact frequency is still given by (A3), with k^H/n^H replacing *k/n*. The contact frequency therefore increases when the economy is opened, leading to higher buyer surplus. But, sellers earn no profits

under free entry, so 'seller surplus' is zero. Total welfare is therefore just equal to the buyer surplus. As claimed in Proposition 2, opening the economy therefore increases welfare when there is free entry.

What about the case where there is not free entry to become a seller, but international trade drive profits down to a degree where some sellers must exit? In that case, international trade also increases buyer surplus: After the sellers have left the markets, the zero profit condition for the open economy will hold. When (A5) holds, λ^H is higher than Home's contact frequency could have been under autarky (because λ^H is higher than the highest possible level of λ for the closed economy).

This completes the proof of Proposition 2.

The number of sellers under free entry in the open economy can be found by combining (5) and (A3) (for both Home and Foreign) and solving for m^H:

$$m^H = \frac{1}{k^H k^F - k_x^H k_x^F}\left[k^F n^H \ln\left(\frac{n^H(\bar{p}-c)}{v'(k^H / n^H)}\right) - k_x^H n^F \ln\left(\frac{n^F(\bar{p}-c)}{v'(k^F / n^F)}\right)\right]. \qquad \text{(A6)}$$

Comparing with (A4), it is ambiguous whether the number of sellers falls or increases when the economies are opened. Import competition tends to squeeze sellers out, but it may be that the domestic price posting expenditure falls sufficiently to allow the number of sellers to increase in both countries. In itself, the number of sellers has no implications for welfare under free entry, in any case sellers earn zero profits. What matters is the total number of buyers reached by their price posting campaigns.

Appendix A.4: *Distributions for quantities and revenues*

The c.d.f. for expected quantities on the domestic market, q, will be

$$F_q(q) = 1 - \frac{1}{\lambda^H}\ln\left(\frac{k^H}{q}\right), \quad \text{support: } q \in \left[\exp\left(-\lambda^H\right)k^H, k^H\right].$$

A similar distribution holds for export quantities, replace λ^H with λ^F and k^H with k_x^H. Sellers' domestic revenue, R, is distributed according to

$$F_R(R) = \frac{1}{\lambda^H}\left(\frac{R}{ck\exp(-\lambda)} - \frac{\bar{p}-c}{c}\right), \text{ support } R \in \left[\bar{p}\exp\left(-\lambda^H\right)k^H, \bar{p}\exp\left(-\lambda^H\right)k^H + ck^H\right].$$

These distributions can all be generalized to the case of vertical quality differences as in section 5.2, but both k^H and the supports would in that case vary with the sellers' quality levels.

References

Anderson, J. and E. Van Wincoop, 2004. Trade Costs. Journal of Economic Literature, Vol. 42, No. 3, pp. 691-751.

Arkolakis, C., 2010. Market Penetration Costs and the New Consumers Margin in International Trade. Journal of Political Economy, No.118 Vol.6, pp.1151-1199.

Armenter, Roc and Miklós Koren. 2009. "Economies of Scale and the Size of Exporters," Central European University, mimeo.

Blas, B. and C.N. Russ, 2011. Understanding Markups in the Open Economy under Bertrand Competition. Unpublished, University of California Davis.

Baye, M., J. Morgan and P. Scholten 2006. Information, Search, and Price Dispersion. In: Handbook of Economics and Information Systems (T. Hendershott, Ed.), Elsevier Press, Amsterdam.

Chaney, T. 2011a. The Network Structure of International Trade. Unpublished, University of Chicago.

Chaney, T. 2011b. The Gravity Equation in International Trade: an Explanation. Unpublished, University of Chicago.

Brander, J., 1981. Intra-industry trade in identical commodities, Journal of International Economics, Vol. 11, pp. 1-14.

Brander, J. and P. Krugman, 1983. A 'Reciprocal Dumping' Model of International Trade. Journal of International Trade, Vol. 15., Is. 3-4, pp. 313-321.

Burdett, K. and K. Judd, 1983. Equilibrium Price Dispersion. Econometrica, Vol. 51 No. 4, pp. 955-959.

Butters, G., 1977. Equilibrium Distributions of Sales and Advertising Prices. The Review of Economic Studies, Vol.44, No.3., pp. 465-491.

Clay, K., R. Krishnan and E. Wolff, 2001. Prices and Price Dispersion on the Web: Evidence from the Online Book Industry. The Journal of Industrial Economics, Vol. 49, No. 4, Symposium on E-Commerce, pp. 521-539.

Cukrowski, J. and E. Aksen, 2003. Perfect competition and intra-industry trade. Economics Letters Vol. 78 , pp. 101-108.

Eaton, J. and S. Kortum, 2002. Technology, Geography, and Trade., Econometrica, Vol.70 pp1741-1779.

Bernard, A.J., J. Eaton, J.B. Jensen, S. Kortum, 2003. Plants and Productivity in International Trade. American Economic Review, Vol.93 pp268-1290.

Eaton, J.,.M. Eslava, M. Kugler and J. Tybout 2007. Export Dynamics in Colombia: Firm-Level Evidence. NBER Working Paper 13531.

Eaton, J., M. Eslava, C. J. Krizan, M. Kugler, and J. Tybout 2011. A Search and Learning Model of Export Dynamics. Unpublished.

Feenstra, R. and M. Shapiro (eds.), 2003. Scanner Data and Price Indexes. University of Chicago Press.

Martin, J. 2010. Markups, Quality and Transport Costs. CREST Working Paper, 2010-17.

Melitz, J. 2008. Language and foreign trade. European Economic Review. Vol. 52, pp. 667-699.

Melitz, M., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity, Econometrica, Vol.71, No.6, pp.1695-725.

Milgrom, P. and J. Roberts, 1986. Price and Advertising Signals of Product Quality. The Journal of Political Economy, Vol. 94, No. 4, pp. 796-821.

Mortensen, D. 1990. Equilibrium Wage Distributions: A Synthesis. In J. Hartog, G. Ridder, and J. Theeuwes, eds., Panel Data and Labor Market Studies. Amsterdam: North Holland.

Mortensen, D., 2003. Wage Dispersion: Why Are Similar Workers Paid Differently? Cambridge, MA: MIT Press.

Munch, J.R. and D.X. Nguyen, 2010. Decomposing Firm-level Sales Variation. Mimeo, University of Copenhagen.

Neary, J.P., 2003. Globalisation and market structure. Journal of the European Economic Association, 1:2-3, April-May 2003, 245-271.

Nelson, P., 1970. Information and Consumer Behavior. The Journal of Political Economy, Vol. 78, No. 2, pp. 311-329.

Nelson, P., 1974. Advertising as Information. The Journal of Political Economy, Vol. 82, No. 4, pp. 729-754.
Portes, R. and H. Rey, 2005. The Determinants of Cross-Border Equity Flows. Journal of International Economics, Vol. 65, Issue 2, pp. 269-296.
Pratt, J., D. Wise, R. Zeckhauser, 1979. Price Difference in Almost Competitive Markets. Quarterly Journal of Economics. Vol. 93, No. 2, pp. 189-211.
Rauch, J. 1999. Networks Versus Markets in International Trade. Journal of International Economics Vol.48, pp7-35.
Rauch, J. 2001. Business and Social Networks in International Trade. Journal of Economic Literature Vol.39 pp1177-1203.
Rauch, J. and V. Trindade, 2002. Ethnic Chinese Networks in International Trade. Review of. Economics and Statistics Vol. 84 No. 1, pp. 116-30.
Rauch, J. and V. Trindade, 2003. Information, International Substitutability, and Globalization. The American Economic Review, Vol. 93, No. 3, pp. 775-791.
Stigler, G. 1961. The Economics of Information. The Journal of Political Economy, Vol. 69, No. 3, pp. 213-225.